TRINITY COLLEGE

MATHEMATICS APPLICATIONS UNIT 3

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Your name

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	5	5	50	51	35
Section Two: Calculator-assumed	10	10	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed   **65% (96 Marks)**
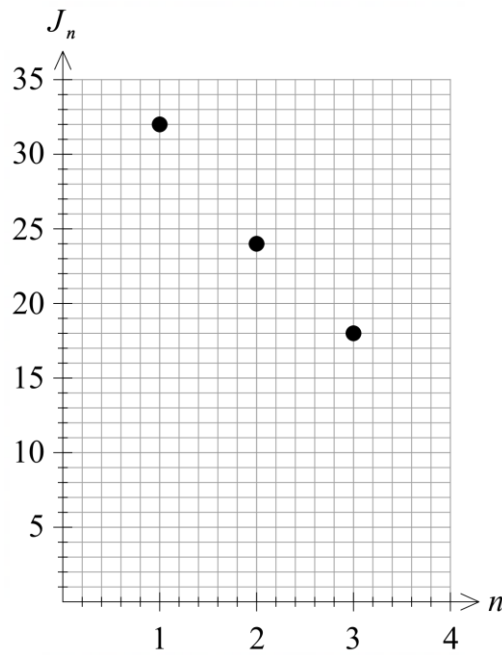
This section has **ten** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 6   **(4 marks)**

The graph below shows the length of a frog's first three jumps, J_n, measured in centimetres.

(a)   Determine the length of the frog's fifth jump.   (2 marks)

To conserve energy, the frog stops jumping once they jump less than 5 cm.

(b)   Determine how many jumps the frog makes.   (2 marks)

See next page

Question 7   **(11 marks)**

Before the last federal election, a polling company asked 1500 people for who they intended to vote. The information is displayed below.

	Voting Intention				
	Labor	**Liberal**	**Independent**	**Other**	**Total**
Male	225			33	713
Female		177	277	22	
Total		382		55	

(a)   Complete the two-way table above.   (2 marks)

(b)   Explain why 'Voting Intention' is not the explanatory variable.   (1 mark)

(c)   An empty two-way percentage table is shown below.

	Voting Intention				
	Labor	**Liberal**	**Independent**	**Other**	**Total**
Male					
Female					
Total					

Complete the table by using either row **or** column percentages, as appropriate.   (3 marks)

(d)   For which candidate is there no association between the intention to vote for them and
        gender?   (1 mark)

(e)   Describe **two** associations that can be observed from the two-way percentage table, giving
        reasons for your answer.   (4 marks)

See next page

Question 8   **(6 marks)**

A connected planar graph has 4 faces and 9 edges.

(a)   Determine the number of vertices for this network.   (2 marks)

(b)   Draw the connected planar network in the space below, labelling the vertices.   (2 marks)

n vertices and n edges are added to the network in part (b), where n is a positive integer.

(c)   If the network is still a planar connected network, does the number of faces change?
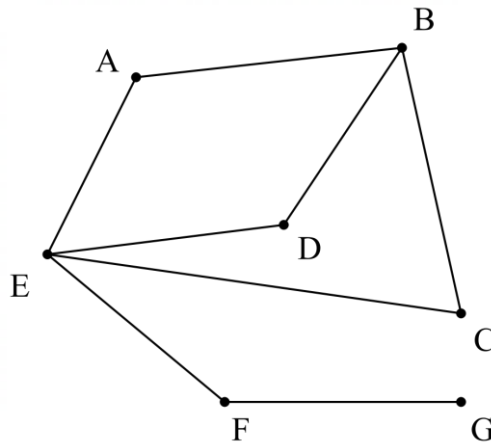   Justify your answer.   (2 marks)

Question 9   **(4 marks)**

Consider the graph below.

(a)   How many bridges does the graph have?   (1 mark)

(b)   Show that the graph is clearly bipartite.   (2 marks)

The bipartite graph splits the vertices into two groups of students. The edges indicate that the two students are friends with each other.

A note is passed from student to student. Only friends can pass it to another friend.

(c) G wants to get a note to C. Explain why E has to be involved.   (1 mark)

**Question 10**   **(12 marks)**

Combustion engines work by converting heat energy into kinetic energy. The efficiency rating, R, is based on the amount of heat energy converted into kinetic energy.

The table below gives the efficiency rating of several engines and the year they were invented. The most efficient an engine can be is when $R = 0$.

Engine	Savery	New-comen	Watt	Cornish	Parsons	Triple	Steam	Gas
Year (t)	1700	1712	1776	1840	1884	1910	1954	1995
Efficiency (R)	-2	-1.9	-1.6	-1.2	-0.7	-0.5	-0.3	-0.2

(a)   Complete the scattergraph below by plotting the data for Steam and Gas engines.   (1 mark)
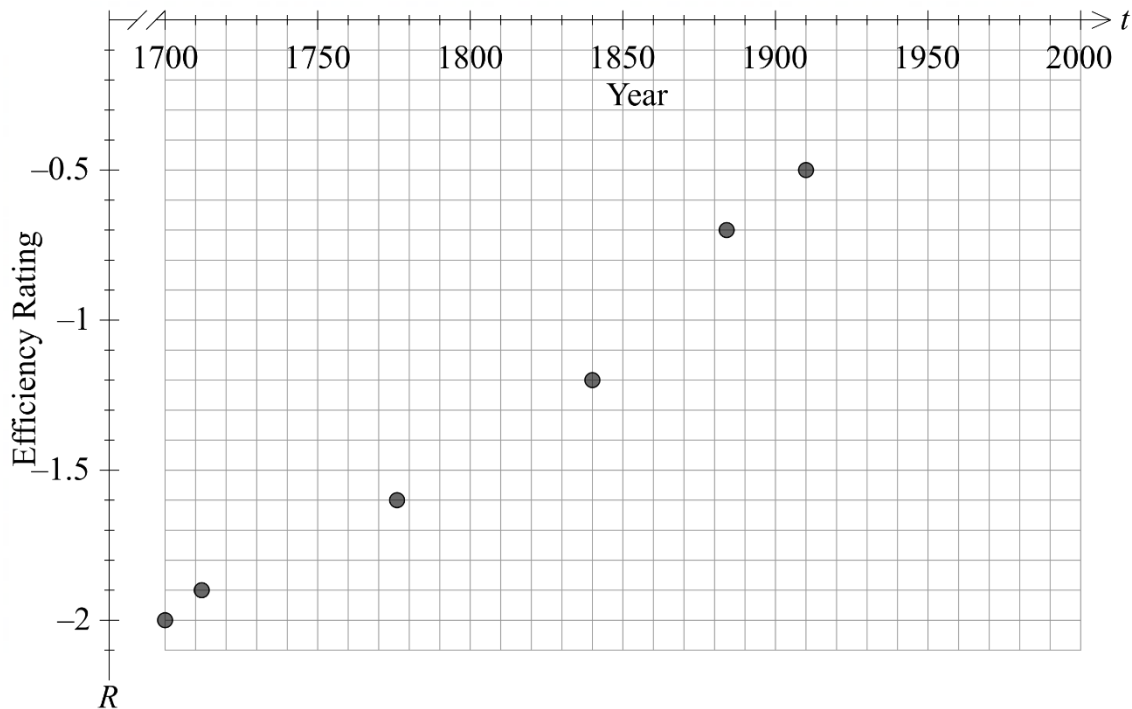
(b)   Calculate the correlation coefficient r_{tR} for this data, correct to four decimal places.   (1 mark)

(c)   Determine the equation of the least-squares line for this data, with coefficients correct to four decimal places.   (2 marks)

See next page

(d)   Plot the least-squares line on the above scattergraph, indicating any points used.   (2 marks)

(e)   (i)   What part of the least-squares line in part (c) indicates that engines are becoming more efficient over time?   (1 mark)

(ii)   Interpret your answer to part (e)(i) in the context of this question.   (1 mark)

A new combustion engine is being developed by a motor company and will be released in 2023.

(f)   (i)   Determine the predicted efficiency rating for this motor.   (1 mark)

(ii)   Comment on the validity of this prediction, giving **two** justifications for your answer.
(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 11   **(16 marks)**

Mark uses mustard in his sandwiches. He places the same amount of mustard in each sandwich.

Mark opened a new jar of mustard. By the end of the fourth day there was 165 g of mustard left in the jar, and at the end of the seventh day there was 120 g of mustard left in the jar.

(a)   Determine the amount of mustard that Mark uses in each sandwich.   (2 marks)

(b)   Determine the amount of mustard in a new jar.   (1 mark)

(c)   Write down a recursive rule for the amount of mustard remaining in the jar, M_n, at the end of nth day.   (2 marks)

(d)   Determine the number of sandwiches that Mark can make from a jar of mustard.   (2 marks)

Mark buys 4 jars of mustard each quarter (every three months). He notices that the cost of his mustard at The Potato Barn increases by 2.3% every quarter. In the first quarter of 2023 he spent $5 on each jar.

(e)   (i)   How much did Mark spend on mustard in the first quarter of 2023?   (1 mark)

(ii)   Hence, write down a recursive rule for the amount that Mark spends on mustard during a quarter, A_n, where A_1 is the first quarter of 2023.   (1 mark)

(f)   Determine how much Mark spends on mustard during 2023.   (2 marks)

(g)   Determine during which quarter and year Mark will start paying $7 a jar.   (3 marks)

Mark finds out that Aldente sells the same mustard for $4.50 a jar in the first quarter of 2023.
However, their prices are increasing by 3.5% each quarter.

(h)   For how long will Aldente be cheaper than The Potato Barn.   (2 marks)

See next page

Question 12   **(10 marks)**

A primary school in a new suburb has capacity for 300 students. The Education Department predicts that each year, the number of new students is 70, and the number of students who leave during the year will be 6.5% of the number of students at the start of the year. Each year, 40 students leave at the end of the year. There are currently 210 students in the primary school.

Let S_n be the number of students in the school at the start of the year, n years from 2023.

The number of students can be represented by the equation

$$S_{n+1} = aS_n + b, \quad S_0 = 210$$

(a)   Explain why $a = 0.935$.   **(1 mark)**

(b)   Determine the value of b.   **(1 mark)**

(c)   Determine the number of students in the primary school at the start of each of the next three years.   **(2 marks)**

The Education Department plans to extend the school in two stages by building new facilities. The first stage will increase capacity by 80 students by the end of 2029.

(d)   Is the completion of the first stage fast enough? Justify your answer.   **(2 marks)**

The second stage will be completed when the school reaches the new capacity of 380, and will increase capacity by a further x students.

(e)   By when should the second stage be completed? Justify your answer.   (2 marks)

The Education Department wishes for the second stage to be big enough to cater for any future growth.

(f)   Determine the value of x, justifying your answer.   (2 marks)

See next page

Question 13 (13 marks)

(a) A survey of surgeons ranked their video game skills (x) with how they performed in simulated surgery (y). The data showed a strong positive correlation. The survey then concluded that playing more video games makes you a better surgeon.

(i) Explain whether the data supports this statement. (2 marks)

(ii) Identify a possible explanation for the association noticed. (2 marks)

(b) A fast-food store owner recorded the daily number of burgers and pizzas sold at their restaurant. The following table and scatter graph shows this data for the last eight days.

Burgers (x)	20	24	25	28	30	30	31	57
Pizzas (y)	31	36	37	43	45	44	45	74
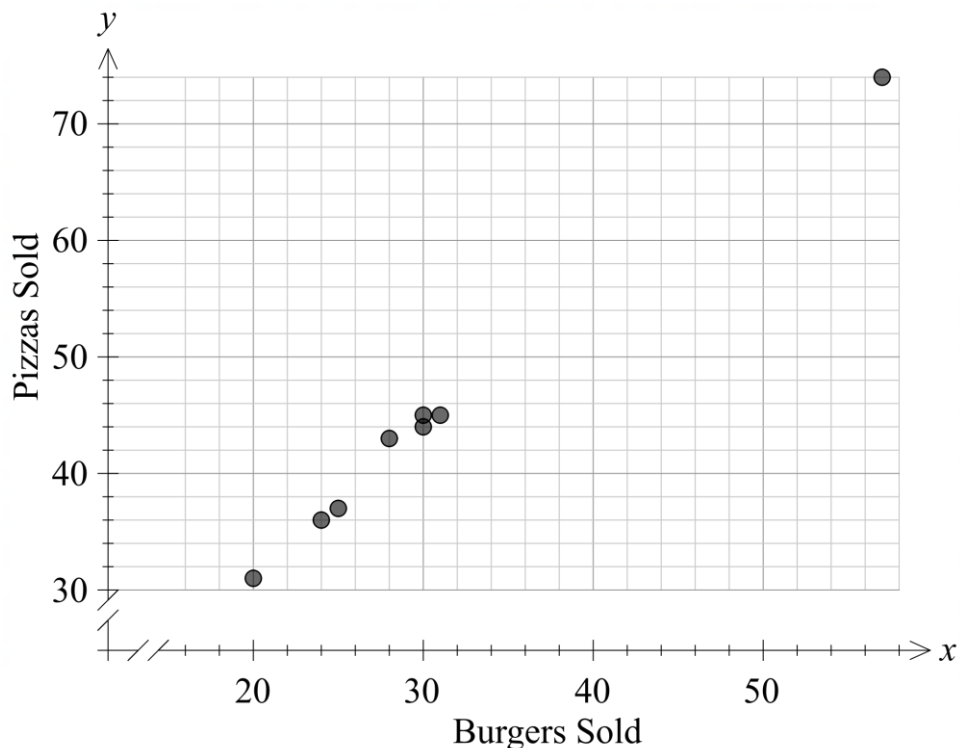
The correlation coefficient of this data is $r_{xy} = 0.9967$.

(i) Comment on the association between the two variables in terms of direction and strength. (2 marks)

The point $(57,74)$ was on a day when there was a festival. The restaurant owner feels that this is an outlier and removes the point. The correlation coefficient is recalculated as $r_{xy} = 0.991$.

(ii) Explain why the correlation coefficient has not changed significantly despite the removal of an outlier. (2 marks)

(iii) Which of the following (**A**, **B** or **C**) would be the least squares regression line for this data.

    A $\hat{y} = 3.78 - 1.35x$ **B** $\hat{y} = 3.78 + 1.35x$ **C** $\hat{y} = 3.78 + 13.5x$

Justify your choice by referring to two features shown on the scatter graph. (3 marks)

The restaurant owner decided to advertise and promote one of their products. The data for another week of sales produced the following:
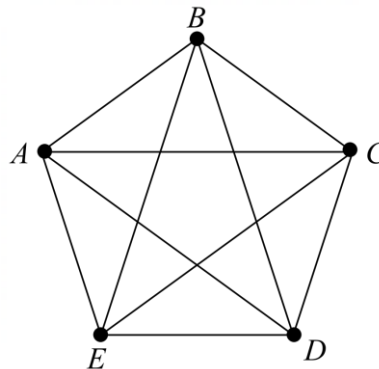
- a least squares regression line of $\hat{y} = 5.76 + 2.71x$
- correlation coefficient of 0.9945

(iv) Determine which product they promoted. Justify your answer by referring to the data above. (2 marks)

See next page

Question 14   **(11 marks)**
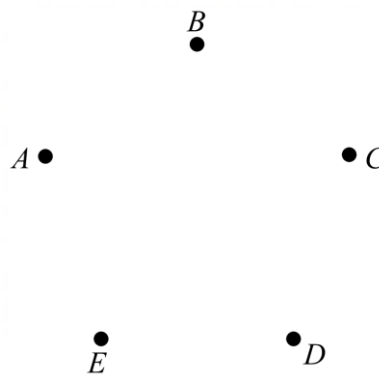
The complete graph with five vertices, K_5, is shown below.

(a)   Explain why K_5 is Hamiltonian.   (1 mark)

(b)   (i)   Identify a Hamiltonian cycle in K_5.   (1 mark)

(ii)   Remove this Hamiltonian cycle from K_5 and draw the resulting subgraph.   (1 mark)

(iii)   Is the subgraph in part (b)(ii) Hamiltonian? Justify your answer.   (2 marks)

See next page

The complete graph with four vertices, K_4, is also Hamiltonian.

(c)   If a Hamiltonian cycle is removed from K_4 is the resulting subgraph Hamiltonian? Explain
       your answer.   (3 marks)

Polish mathematician, Kasimir Kuratowski, proved that if K_5 was a subgraph of a larger graph, then
the larger graph cannot be planar.

(d)   Show that the graph below is not planar, by drawing a suitable subgraph below.   (3 marks)

Question 15 (9 marks)
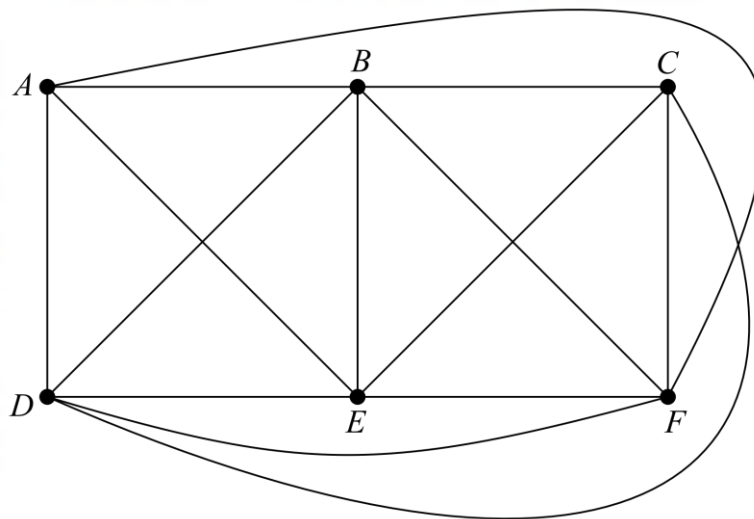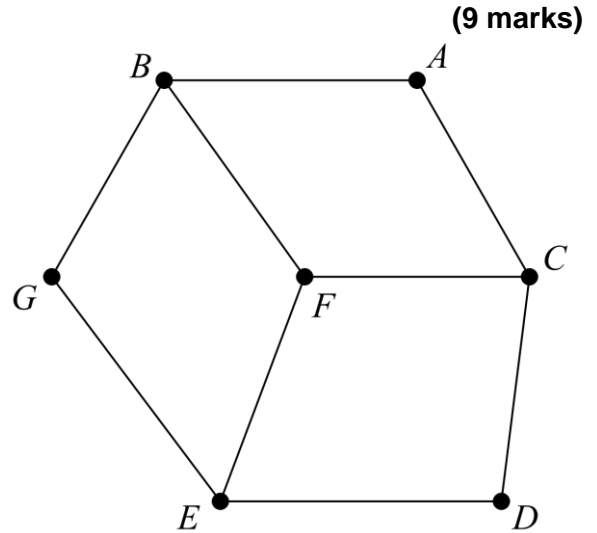
The diagram shows the air conditioning ducting
installed in the roof of a warehouse.

A tradesperson is employed to inspect the ducting.

They must enter the roof through a manhole at B.

The tradesperson needs to inspect each section of
ducting at least once and wishes to minimise the
number of sections inspected more than once.

(a) (i) Explain why the tradesperson will need to inspect some sections more than once.
(2 marks)

(ii) Suggest a route by which the tradesperson can inspect each section of ducting,
minimising the number of sections inspected more than once. State the sections which
are inspected more than once. (3 marks)

(b) A new piece of ducting is added between C and E. What effect will this have on the route you
found in part (a)(ii)? Justify your answer. (3 marks)

(c) What feature of the graph indicates that vertex F must be visited twice? (1 mark)

End of questions

Supplementary page

Question number: _____